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FUZZY IDEALS OF BBG-ALGEBRAS

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Abstract

In this work, we explored various related characteristics and introduced fuzzy ideals and fuzzy congruences in BBG-algebras. We also examine the relationship on commutative BBG-algebras between fuzzy ideals and fuzzy congruences. We characterized and investigated various characteristics of the fuzzy quotient in commutative BBG-algebras generated by this relation

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Keywords: BBG-algebras, fuzzy congruences, fuzzyideals, fuzzy quotient BBG-algebras.

1. Introduction

An ideals of in BCC-algebras was presented by Dudek and Zhang [4], who also discussed the relationships between these ideals and congruences. Ding and Pang [5] gave the definition, characteristics, and use of quotient algebras as a fundamental tool for examining the structures of BCI-algebras. Prabpayak and Leerawat [6] presented KU-algebras, a type of algebra in which ideals are defined and congruences on KU-algebras are examined.
Asawasamrit and Leerawat [2] analyzed the relationship between ideals and congruences and proposed an algebraic structure known as a binary algebra. They also looked at the properties of quotient binary algebra and described it.
Asawasamrit and Sudprasert [3] established the general theory of KK-algebras and demonstrated the relationship between ideals and congruences.

The first time fuzzy sets and fuzzy relations were proposed b L.A. Zadeh [1]. Numerous writers then conducted research on it
K.J. Lee [9] presented ideals in pseudo BCI-algebras. Fuzzy ideals of pseudo BCK-algebras proposed by Dymek, Walendziak [10]. Also, Murali [11, 12] studied fuzzy congruence relations on algebras.

Further, M. Kondo [13] defined a fuzzy congruence relation on a group and showed that there is a lattice isomorphism between the set of fuzzy normal subgroups of a group and the set of fuzzy congruencyon this group. Recently, A. Rezaei et al [14, 15, 16] discussed on (fuzzy) congruence relations in (pseudo) CI/BE-algebras and studied some oftheir properties.

In the present study, we studied certain related characteristics and presented the ideas of fuzzy ideals and fuzzy congruences on BBG-algebras. Additionally, we looked at the relationship on commutative BBG-algebras between fuzzy ideals and fuzzy congruences. We characterized and investigated the features of the fuzzy quotient in commutative BBG-algebras generated by this relation.

2. PRELIMINARIES

In this topic, start with basic axioms and important results which we need in the sequel.

Definition 2.1. [7] A BBG-algebra is algebra $(H,*,0)$ with a binary operation $*$, for every $\alpha,\beta,\gamma,\in H$ which satisfies the following properties:

(1) $\alpha * \alpha = 0,$
(2) $0 * \alpha = \alpha,$
(3) $\alpha * (\beta * \gamma) = ((\beta * 0) * \alpha) * \gamma.$

Theorem 2.2. [7] Let $(H,*,0)$ be a BBG-algebra. Then, for every $\alpha,\beta,\gamma,\in H$:

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(1) $((\alpha * 0) * (\alpha * \beta) = \beta$,
(2) $\alpha * \beta = \alpha * \gamma \Rightarrow \beta = \gamma$,
(3) $(\alpha * \beta) * \gamma = \beta * ((\alpha * 0) * \gamma)$,
(4) $\alpha * \beta = 0 \Rightarrow \alpha = \beta$,
(5) $(\alpha * 0) * 0 = \alpha$,
(6) $(\beta * \alpha) * 0 = \alpha * \beta$,
(7) $\beta * \alpha = \gamma * \alpha \Rightarrow \beta = \gamma$.

Definition 2.3. [7] A commutative BBG-algebra (CBBG-algebra) is a BBG-algebra with the characteristic that $(\alpha * 0) * \beta = (\beta * 0) * \alpha$ for every $\alpha, \beta, \gamma, \in H$. Then a binary operation $\wedge$ on a commutative BBG-algebra defined on $(H, *, 0)$ by $\alpha \wedge \beta = (\alpha * \beta) * \beta$ and showed that $\alpha \wedge \beta = \beta \wedge \alpha$ for all $\alpha, \beta \in H$ and proved the following proposition.

Theorem 2.4. [7] If $(H, *, 0)$ is a commutative BBG-algebra, then for every $\mu, \pi, \alpha, \beta, \gamma, \in H$:

(1) $(\beta * 0) * (\alpha * 0) = \alpha * \beta$,
(2) $(\alpha * \beta) * \beta = \alpha$,
(3) $(\beta * \pi) * (\alpha * \mu) = (\mu * \pi) * (\alpha * \beta)$,
(4) $(\gamma * \alpha) * (\gamma * \beta) = \alpha * \beta$,
(5) $(\beta * \gamma) * (\alpha * \gamma) = \alpha * \beta$,
(6) $(\pi * \beta) * \alpha = (\alpha * \beta) * \pi$,
(7) $\alpha * (\beta * \gamma) = \beta * (\alpha * \gamma)$.

Lemma 2.6. [8] Let A is a closed subset of a BBG-algebra $(H, *, 0)$. Then $0 \in A$.

Definition 2.7. [8] Let $(H, *, 0)$ be a BBG-algebra and A be a subsetof H. Then A is known as an ideal of $(H, *, 0)$ under the operation, if it satisfies the following conditions

(1) $0 \in A$,

(2) For $\alpha, \beta \in H$, if $\alpha \in A$ and $\alpha * \beta \in A$ then $\beta \in A$.

3. Fuzzy ideals BBG-algebras

In the present study, we study closed fuzzy subsets (CFS), fuzzy ideals (FI) and closed fuzzy ideals (CFI) and their properties.

Definition 3.1. Let $(H, *, 0)$ be a BBG-algebra and f be a non-empty fuzzy subset of H. Then, f is CFS of H if $f(0) = 0$ and $f(\alpha) \wedge f(\beta) \leq f(\alpha * \beta)$ for every $\alpha, \beta \in H$.

We can characterize fuzzy closed subset by using the level closed subset of BBG-algebras.

Theorem 3.2. Let f be a fuzzy set in H. Then, f is a CFS of H iff, for every $a \in [0,1], f_a$ is a closed set of H.

Proof. Suppose that f is a fuzzy set in H, then $f(0) = 0$. This implies $0 \in f_\alpha$ and so $f_\alpha \neq \emptyset$. Let $\alpha, \beta \in \xi_a$. Then, $f(\alpha * \beta) \geq f(\alpha) \wedge f(\beta) \geq a$. Hence $\alpha * \beta \in f_a$.

Conversely, suppose that $a \in [0,1], f_a$ is a closed subset of H. Since

$0 \in f_a$ for every $a \in [0,1]$. Put $a = 1$, then $0 \in f_1$.

This implies $f(0) \geq f(1)$. Hence $f(0) = 1$.

Also, let $f(\alpha) = \alpha_1$ and $f(\beta) = \alpha_2$. This implies $\alpha \in \xi_{\alpha_1}$ and $\beta \in \xi_{\alpha_2}$.

Put $\alpha = \alpha_1 \wedge \alpha_2$, then $\alpha \leq \alpha_1$ and $\alpha \leq \alpha_2$. This indicates $\alpha \in \xi_{\alpha_1} \subseteq \xi\alpha$ and $\beta \in \xi_{\alpha_2} \subseteq \xi_\alpha$. This implies $\alpha\beta \in \xi_\alpha$, and so $\xi(\alpha * \beta) \geq \alpha = \alpha_1 \wedge \alpha_2 = \xi(\alpha) \wedge \xi(\beta)$.

Corollary 3.3 Let $\phi \neq I \subseteq H$. Then, H is an closed subset iff χ_I is a CFS of H.

Definition 3.4. Let $(H, *, 0)$ be a BBG-algebra and f be a fuzzy subset of H. Then f is called a FI of $(H, *, 0)$ under the operation $*$, if it satisfies the following conditions:

(1) $f(0) = 0$,

(2) $f(\alpha) \wedge f(\alpha * \beta) \leq f(\beta)$ for every $\alpha, \beta \in H$.

We can characterize fuzzy ideals by using the level ideals of an BBG-algebras.

Theorem 3.5. Let f be a fuzzy set in H. Then, f is a FI of H if and only if, for all $a \in [0,1], f_a$ is an ideal of H.

Proof. Suppose that f is a FI in H, then $f(0) = 0$. This implies $0 \in f_\alpha$ and so $f_\alpha \neq \emptyset$. Let $\alpha, \alpha * \beta \in f_a$. Then, $f(\beta) \geq f(\alpha) \wedge f(\alpha * \beta) \geq a$. Hence $\beta \in f_a$.

Conversely, suppose that $a \in [0,1], f_a$ is a fuzzy ideal of H.

Since $0 \in f_a$ for every $a \in [0,1]$. Put $a = 1$, then $0 \in f_1$.

This implies $f(0) \geq f(1)$.

Hence $f(0) = 1$.

Also, let $f(\alpha) = a_1$ and $f(\alpha * \beta) = a_2$. This implies $a \in f_{a_1}$ and $\alpha * \beta \in f_{a_2}$. Put $\alpha = \alpha_1 \wedge \alpha_2$, then $a \leq a_1$ and $a \leq a_2$. This indicates $\alpha \in f_{a_1} \subseteq f_a$ and $\alpha * \beta \in f_{a_1} \subseteq f_a$. This implies $\beta \in f_a$, and so $f(\beta) \geq a = a_1 \wedge a_2 = f(\alpha) \wedge f(\alpha * \beta)$.

Corollary 3.6. Let $\phi \neq I \subseteq H$. Then, I is an ideal iff χ_I is a FI of H.

Example 3.7. Let $H = \{0, 1, 2, 3, 4, 5\}$ and let the binary operation $*$ be defined by table 1.

$*$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	2	0	1	4	5	3
2	1	2	0	5	3	4
3	3	4	5	0	1	2
4	4	5	3	2	0	1
5	5	3	4	1	2	0

Table 1

We see in [7], $(H,*,0)$ is a BBG-algebra. Define a fuzzy subset f, h and i on H as follows: $f(0) = 1, f(1) = f(2) = 0.6$ and $f(3) = f(4) = f(5) = 0.2$. $h(0) = h(3) = 1, h(1) = h(2) = 0.8$ and $h(4) = h(5) = 0.3$, and $i(0) = i(0) = 1$ and $i(1) = i(2) = i(3) = i(4)$ are CFI of H.

Example 3.8. Let $\{0,1,2,3\}$ and let $*$ the operation defined by the table-2.

$*$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Table 2

Lemma 3.9. Let $(H,*,0)$ be CBBG-algebra and let f bea CFS of H. Then f is a FI of f if and only if $f(\alpha) \wedge (1 - f(\gamma * \beta)) \geq (1 - f(\gamma * (\alpha * \beta)))$ for all for every $\alpha, \beta, \gamma \in H$.

Proof. Suppose that f_i is a CFI of X. Then, for any $\alpha, \beta, \gamma \in H$, $f(\alpha) \wedge f(\gamma * \beta) \leq f(\alpha * (\gamma * \beta)) = f(\gamma * (\alpha * \beta))$. This implies

$$f(\alpha) \wedge \big(1 - f(\gamma * \beta)\big) = 1 - \big(f(\alpha) * f(\gamma * \beta)\big) \geq 1 - \Big(f\big(\gamma * (\alpha * \beta)\big)\Big).$$

Conversely, since f is a CFS of H, then $f(\alpha) \leq f(0)$ for all $\alpha \in H$.
Suppose that $f(\alpha) \wedge 1 - f(\gamma * \beta)) \leq (1 - f(\gamma * (\alpha * \beta)))$ for any $\alpha, \beta, \gamma \in H$. This implies that $f(\alpha) \wedge f(\gamma * \beta)) \leq f(\gamma * (\alpha * \beta)))$ for any $\alpha, \beta, \gamma \in H$.
Put $\gamma = \beta$, we have that $f(\alpha) \wedge f(\alpha * \beta)) \leq f(\alpha * (\alpha * \beta))) = f((\alpha * \alpha) * \beta)) = f(0 * \beta)) = f(\beta)$ for any $\alpha, \beta, \gamma \in H$. This implies f is a fuzzy ideal of H.
We see in [7], $(H,*,0)$ is a CBBG-algebra.
Define a fuzzy subset f on H as follows: $f(0) = 1 = f(2)$, and $f(1) = f(3) = 0.6$ are CFI of H.

Lemma 3.9. Let $(X,*,0)$ be a CBBG-algebra and let f bea CFS of X. Then f is a FI of X if and only if $f(\alpha) \wedge (1 - f(\gamma * \gamma)) \geq (1 - f(\gamma * (\alpha * \beta)))$ for all for any $\alpha, \beta, \gamma \in H$.

Proof. Suppose that f_i is a CFI of H. Then, for any $\alpha, \beta, \gamma \in H$., $f(\alpha) \wedge f(\gamma * \beta) \leq f(\alpha * (\gamma * \beta)) = f(\gamma * (\alpha * \beta))$. This implies

$$f(\alpha) \wedge \big(1 - f(\gamma * \beta)\big) = 1 - \big(f(\alpha) * f(\gamma * \beta)\big) \geq 1 - \Big(f\big(\beta * (\alpha * \beta)\big)\Big).$$

Conversely, since f is a CFS of H, then $f(\alpha) \leq f(0)$ for any $\alpha \in H$.
Suppose that $f(\alpha) \wedge 1 - f(\gamma * \beta)) \leq (1 - f(\gamma * (\alpha * \gamma)))$ for any $\alpha, \beta, \gamma \in H$.. This implies that $f(\alpha) \wedge f(\gamma * \beta)) \leq f(\gamma * (\alpha * \beta)))$ for any
Put $\gamma = \beta$, we have that $f(\alpha) \wedge f(\alpha * \beta)) \leq f(\alpha * (\alpha * \beta))) = f((\alpha * \alpha) * \beta)) = f(0 * \beta)) = f(\beta j)$ for any $\alpha, \beta, \gamma \in H$. This implies ξ is a FI of H.

Corollary 3.10. Let f be a CFS of a CBBG-algebra H. Then f is a FI of H if and only if $f(\alpha) \wedge (1 - f(\beta)) \geq (1 - f(\alpha * \beta))$ for any $\alpha, \beta, \gamma \in H$.

Lemma 3.11. Let ξ be a CFS of a CBBG-algebra H. Then f is a FI of H if and only if $f(\alpha * (\beta * \gamma)) \wedge (1 - f(\alpha * \gamma)) \geq (1 - f(\beta))$ for any $\alpha, \beta, \gamma \in H$.

Corollary 3.12. Let ξ be a CFS of a CBBG-algebra H. Then f is a FI of Hif and only if $f(\alpha * \beta)) \wedge (1 - f(\beta)) \geq (1 - f(\alpha))$ for any $\alpha, \beta, \gamma \in H$.

Theorem 3.13. Let $\{f_t : t \in \mathbb{N}\}$ be a family of FI in a BBG-algebra H, where $f_n \subseteq f_{n+1}$ for any $n \in \mathbb{N}$. Then $\bigcup_{n=1}^{\infty} f_n(y)$ is a FI of BBG-algebra of H.

Proof. Since f_i is a FI, then $\xi_1(0) = 1$ and $f_1 \subseteq \bigcup_{n=1}^{\infty} f_n$. This implies $\bigcup_{n=1}^{\infty} f_n(0) = 1$.
Also, $\bigcup_{n=1}^{\infty} f_n(\alpha) \wedge \bigcup_{n=1}^{\infty} f_n(\alpha * \beta) = \bigvee_{n=1}^{\infty}\{f_t(\alpha) : t \in (1,\infty)\} \wedge \bigvee_{n=1}^{\infty}\{f_r(\alpha * \beta) : r \in (1,\infty)\} = \bigvee_{n=1}^{\infty}\{f_t(\alpha) \wedge \xi_r(\alpha * \beta) : t, r \in (1,\infty)\}$.
Without generality $r \leq t$ such that $f_r \subseteq f_t$. This implies $\bigvee_{n=1}^{\infty}\{f_t(\alpha) \wedge f_r(\alpha * \beta) : t, r \in (1,\infty)\} \leq \bigvee_{n=1}^{\infty}\{f_t(\alpha) \wedge f_t(\alpha * \beta) : t \in (1,\infty)\} \leq \bigvee_{n=1}^{\infty}\{f_t(\beta) : t \in (1,\infty)\} = \bigcup_{n=1}^{\infty} f_n(\beta)$.
This implies $\bigcup_{n=1}^{\infty} f_n(j)$ is a FI of BBG-algebra of H.

Theorem 3.13. Let $\{f_t : t \in \mathbb{N}\}$ be a family of FIs in a BBG-algebraH, where $f_n \subseteq f_{n+1}$ for any $n \in \mathbb{N}$. Then $\bigcup_{n=1}^{\infty} f_n(\beta)$ is a CFI of BBG-algebra of H.

Proof: For any $\alpha, \beta \in H$, $\bigcup_{n=1}^{\infty} f_n(i) \wedge \bigcup_{n=1}^{\infty} f_n(\beta) = \bigvee_{n=1}^{\infty}\{f_t(\alpha) : t \in (1,\infty)\} \wedge \bigvee_{n=1}^{\infty}\{f_r(\beta) : r \in (1,\infty)\} = \bigvee_{n=1}^{\infty}\{f_t(\alpha) \wedge$

$f_r(\beta) : t, r \in (1,\infty)\}$. Without generality $r \leq t$ such that $f_r \subseteq f_t$.

This implies $\bigvee_{n=1}^{\infty}\{f_t(\alpha) \wedge f_r(\beta) : t, r \in (1,\infty)\} \leq \bigvee_{n=1}^{\infty}\{f_t(\alpha) \wedge f_t(\beta) : t \in (1,\infty)\} \leq \bigvee_{n=1}^{\infty}\{f_t(\alpha * \beta) : j \in (1,\infty)\} = \bigcup_{n=1}^{\infty} f_n(\alpha * \beta)$.

This implies $\bigcup_{n=1}^{\infty} \xi_n(\beta)$ is a CFS of BBG-algebra of H.

Also, by Theorem 3.13, $\bigcup_{n=1}^{\infty} f_n(\beta)$ is a FI of BBG-algebra of H. Hence $\bigcup_{n=1}^{\infty} f_n(\beta)$ is a CFI of BBG-algebra of X.

Theorem3.14: Let $\{f_t : t \in I\}$ be a family of FIs in a BBG-algebra H. Then $\bigcap_{t\epsilon I} f_t$ is a FI of H.

Proof: $\bigcap_{t\epsilon I} f_t(0) = \wedge\{f_t(0) : \forall t \in I\} = 1$.

For any $\alpha, \beta \in H$, $\bigcap_{t\epsilon I} f_t(\alpha) \wedge \bigcap_{t\epsilon I} f_t(\alpha*\beta) = \wedge\{f_t(\alpha) : \forall t \in I\} = \wedge\{f_t(\alpha*\beta) : \forall i \in I\} = \wedge\{f_t(\alpha) \wedge f_t(\alpha*\beta) : \forall t \in I\} \leq \wedge\{f_t(\beta) : \forall t \in I\} = \bigcap_{t\epsilon I} f_t(\beta)$. Hence $\bigcap_{t\epsilon I} f_t$ is a FI of H.

Theorem3.15: Let $\{f_t : t \in I\}$ be a family of CFIs in a BBG-algebra H. Then $\bigcap_{t\epsilon I} f_t$ is a CFI of H.

Proof: For any $\alpha, \beta \in H$, $\bigcap_{t\epsilon I} f_t(\alpha) \wedge \bigcap_{t\epsilon I} f_t(\beta) = \wedge\{f_t(\alpha) : \forall t \in I\} \wedge \{f_t(\beta) : \forall t \in I\} \leq \wedge\{f_t(\alpha*\beta) : \forall t \in I\} = \bigcap_{t\epsilon I} f_t(\alpha*\beta)$. Hence $\bigcap_{t\epsilon I} f_t$ is a CFI of H.

By using above and by Theorem 3.15, $\bigcap_{t\epsilon I} f_t$ is a CFI of H.

4. Fuzzy Congruence Of BBG-Algebras

In this section, we study fuzzy congruence (FC) and fuzzy quotient (FQ) on BBG-algebras.

Definition 4.1. A fuzzy relation θ is a FC of BBG-algebra C if it is a fuzzy equivalence relation X and fuzzy compatibleof H.

Definition 4.2. Let f be a FI of a BBG-algebra H. Define a fuzzy relation θ on H by $\theta(\alpha,\beta) = f(\alpha*\beta) \wedge f(\beta*\alpha)$ for all $\alpha \in H$.

Theorem4.3. If f is a FI of a CBBG-algebra H, then fuzzy relation θ on H is an equivalence relation on H.

Proof. For any$\alpha \in H$, $\theta(\alpha,\alpha) = f(\alpha*\alpha) \wedge f(\alpha*\alpha) = f(0) = 1$ Thisimplies θ is fuzzy reflexive.

For any $\alpha,\beta \in H$, $\theta(\alpha,\beta) = f(\alpha*\beta) \wedge f(\beta*\alpha) = f(\beta*\alpha) \wedge f(\alpha*\beta) = \theta(\beta,\alpha)$. This implies θ is fuzzy symmetric. Next, we show that θ is fuzzy transitive.

For any $\alpha,\beta,\gamma \in H$,

$\theta(\alpha,\beta) \wedge \theta(\beta,\gamma) = f(\alpha*\beta) \wedge f(\beta*\alpha) \wedge f(\beta*\gamma)$ $f(\gamma*\beta)$

$\leq f(\beta*\alpha) \wedge f(\gamma*\beta$

$= f((\gamma*\beta)*(\gamma*\alpha)$

$\wedge f(\gamma*\beta) \leq f(\gamma*\alpha)$.

Similarly $\theta(\alpha,\beta) \wedge \theta(\beta,\gamma) \leq f(\alpha*\beta)$.

This implies $\theta(\alpha,\beta) \wedge \theta(\beta,\gamma) = f(\alpha*\gamma) \wedge f(\gamma*\alpha) = \theta(\alpha,\gamma)$.

This implies θ is a fuzzy transitive. Hence θ is a fuzzy equivalencerelation.

Lemma 4.4. Let f is a FI of a CBBG-algebra H, then $\theta(u,v) \wedge \theta(\alpha,\beta) \leq \theta(u*\alpha, v*\beta)$ for any $\alpha,\beta,u,v \in H$.

Proof. $\theta(u,v) \wedge \theta(\alpha,\beta) = f(u*v) \wedge f(v*u) \wedge f(\alpha*\beta) \wedge f(\alpha*\beta)$

$\leq f(u*v) \wedge f(v*u)$

$= f((v*\alpha)*(u*\alpha) \wedge f((u*\alpha)*(v*\alpha)$

$= \theta(u*\alpha, v*\alpha)$.

$\theta(u,v) \wedge \theta(\alpha,\beta) = f(u*v) \wedge f(v*u) \wedge f(\alpha*\beta) \wedge f(\beta*\alpha)$

$\leq f(\alpha*\beta) \wedge f(\beta*\alpha)$

$= f((v*\beta)*(v*\alpha) \wedge f((v*\beta)*(v*\alpha)$

$= \theta(v*\alpha, v*\beta)$.

By Theorem 4.3,$\theta(u,v) \wedge \theta(\alpha,\beta) \leq \theta(u*\alpha, v*\alpha) \wedge \theta(v*\alpha, v*\beta) \leq \theta(u*\alpha, v*\beta)$.

Corollary 4.5. If f is a FI of a CBBG-algebra H, then the fuzzy relation θ is a fuzzy congruence relation on H.

Proof. By Theorem 4.3 and 4.4, the fuzzy relation θ is a FC relation on H.

Definition 4.6. Let f be a FI of a CBBG-algebraH. The fuzzy sub set f_i on H defined as $f_i(\beta) =$

$f(\alpha*\beta)$ for $\alpha,\beta \in H$. If $f_\alpha = f_\beta$ if and only if $f(\alpha*\beta) = 1$ and $f(\beta*\alpha) = 1$, $(\xi_\alpha = \xi_\beta \Leftrightarrow \theta(\alpha,\beta) = 1)$. It is call to be fuzzy equivalence class ξ_α of α. We defined a fuzzy quotient set by $H/f = \{f_\alpha : \alpha \in H\}$.

Theorem 4.7. Let f be a FI of a CBBG-algebra $(H,*,0)$. Then $(H/\xi, \circ, \xi_0)$ is a commutative BBG-algebra the operation $\circ$ on H/ξ defined by $f_\alpha \circ f_\beta = f_{\alpha*\beta}$ for all $f_\alpha, f_\beta \in H/\xi$.

Proof. We prove that $(H/f, \circ, f_0)$ is a BBG-algebra.

(1) For any $\alpha,\beta,\gamma,\delta \in H$ and for any f_α, f_β, f_γ, $\xi_\delta \in H/\xi$ such that f_α=f_β and f_γ=f_δ. We claim ∘ that $f_\alpha \circ f_\gamma$=$f_\beta \circ f_\delta$. If f_α=f_β, then $\theta(\alpha,\beta) = 1$, and if f_γ=f_δ, then $\theta(\gamma,\delta) = 1$. This implies

$1 = \theta(\alpha,\beta) \wedge \theta(\gamma,\delta) \leq \theta((\alpha*\gamma),(\beta*\delta))$ and

This implies

$$1 = \theta((\alpha * \gamma), (\beta * \delta)).$$

This implies $\xi_{\alpha*\gamma} = \xi_{\beta*\delta}$.

Thus

$$\xi_\alpha \circ \xi_\gamma = \xi_{\alpha*\gamma} = \xi_{\beta*\delta} = \xi_\gamma \circ \xi_\delta.$$

(2) For any $\xi_\alpha, \xi_\beta, \xi_\gamma$ and $\xi_\delta \in H/\xi$

(3) $\xi_\alpha \circ \xi_\alpha = \xi_{\alpha*\alpha} = \xi_0$

(4) $\xi_\alpha \circ (\xi_\beta \circ \xi_\gamma) = \xi_\alpha \circ (\xi_{\beta*\gamma}) = \xi_{\alpha*(\beta*\gamma)} = \xi_{[((\beta*0)*\alpha)*z]} = \xi_{((\beta*0)*\alpha)} \circ \xi_\gamma = (\xi_{(\beta*0)} \circ \xi_\alpha) \circ \xi_\gamma = ((\xi_\beta \circ \xi_0) \circ \xi_\alpha) \circ \xi_\gamma$.

We prove that $(H/f, \circ, f_0)$ is a CBBG-algebra. $(f_i \circ \xi_0) \circ f_j = f_{i*0} \circ f_j = f_{(i*0)*j} = f_{(j*0)*i} = ((f_j \circ f_0) \circ f_i$.

Therefore, $(H/f, \circ, f_0)$ is a CBBG-algebra.

The set H/f is called the FQ CBBG-algebra

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